

CS271 Homework 3 Solution

10-2-24

This is the complete bipartite graph $K_{2,4}$. The vertices in the part of size 2 are c and f , and the vertices in the part of size 4 are a , b , d , and e .

10-2-28

a) Following the lead in Example 14, we construct a bipartite graph in which the vertex set consists of two subsets - one for the employees and one for the jobs. Let $V_1 = \text{Zamora, Agraharam, Smith, Chou, Macintyre}$, and let $V_2 = \text{planning, publicity, sales, marketing, development, industry relations}$. Then the vertex set for our graph is $V = V_1 \cup V_2$. Given the list of capabilities in the exercise, we must include precisely the following edges in our graph: Zamora, planning, Zamora, sales, Zamora, marketing, Zamora, industry relations, Agraharam, planning, Agraharam, development, Smith, publicity, Smith, sales, Smith, industry relations, Chou, planning, Chou, sales, Chou, industry relations, Macintyre, planning, Macintyre, publicity, Macintyre, sales, Macintyre, industry relations.

b) Many assignments are possible. If we take it as an implicit assumption that there will be no more than one employee assigned to the same job, then we want a maximum matching for this graph. So we look for five edges in this graph that share no endpoints. A little trial and error gives us, for example, Zamora, planning, Agraharam, development, Smith, publicity, Chou, sales, Macintyre, industry relations. We assign the employees to the jobs given in this matching.

10-4-14

c) The cycle $abcd fghia$ guarantees that these eight vertices are in one strongly connected component. Since there is no path from e to any other vertex, this vertex is in a strong component by itself. Therefore the strongly connected components are a, b, c, d, f, g, h, i and e .

10-4-16

The given conditions imply that there is a path from u to v , a path from v to u , a path from v to w , and a path from w to v . Concatenating the first and third of these paths gives a path from u to w , and concatenating the fourth and second of these paths gives a path from w to u . Therefore u and w are mutually reachable.

10-5-2

All the vertex degrees are even, so there is an Euler circuit. We can find one by trial and error, or by using Algorithm 1. One such circuit is $a, b, c, f, i, h, g, d, e, h, f, e, b, d, a$.