CS271 Homework 1 Solution

1-1-14

a) $r \wedge \neg q$ **b)** $p \wedge q \wedge r$ **c)** $r \rightarrow p$ **d)** $p \wedge \neg q \wedge r$ **e)** $(p \wedge q) \rightarrow r$ **f)** $r \leftrightarrow (q \vee p)$

1-1-38

p	q	r	s	$p \rightarrow q$	$(p \rightarrow q) \rightarrow r$	$((p \rightarrow q) \rightarrow r) \rightarrow s$
0	0	0	0	1	0	1
0	0	0	1	1	0	1
0	0	1	0	1	1	0
0	0	1	1	1	1	1
0	1	0	0	1	0	1
0	1	0	1	1	0	1
0	1	1	0	1	1	0
0	1	1	1	1	1	1
1	0	0	0	0	1	0
1	0	0	1	0	1	1
1	0	1	0	0	1	0
1	0	1	1	0	1	1
1	1	0	0	1	0	1
1	1	0	1	1	0	1
1	1	1	0	1	1	0
1	1	1	1	1	1	1

1-3-18

It is easy to see from the definitions of the conditional statement and negation that each of these propositions is false in the case in which p is true and q is false, and true in other three cases. Therefore the two propositions are logically equivalent.

1-3-32

We just need to find an assignment of truth values that makes one of these propositions true and the other false. We can let p be true and the other two variable be false. Then the first statement will be $F \rightarrow F$, which is true, but the second will be $F \wedge T$, which is false.

1-4-10

a) $\exists x(C(x) \wedge D(x) \wedge F(x))$
b) $\forall x(C(x) \vee D(x) \vee F(x))$
c) $\exists x(C(x) \wedge F(x) \wedge \neg D(x))$

d) $\neg\exists x(C(x) \wedge D(x) \wedge F(x))$
e) Here the owners of these pets can be different: $(\exists x C(x)) \wedge (\exists x D(x)) \wedge (\exists x F(x))$. There is no harm in using the same dummy variable, but this could also be written, for example, as $(\exists x C(x)) \wedge (\exists y D(y)) \wedge (\exists z F(z))$.

1-4-18
a) $P(-2) \vee P(-1) \vee P(0) \vee P(1) \vee P(2)$
b) $P(-2) \wedge P(-1) \wedge P(0) \wedge P(1) \wedge P(2)$
c) $\neg P(-2) \vee \neg P(-1) \vee \neg P(0) \vee \neg P(1) \vee \neg P(2)$
d) $\neg P(-2) \wedge \neg P(-1) \wedge \neg P(0) \wedge \neg P(1) \wedge \neg P(2)$
e) $\neg(P(-2) \vee P(-1) \vee P(0) \vee P(1) \vee P(2))$
e) $\neg(P(-2) \wedge P(-1) \wedge P(0) \wedge P(1) \wedge P(2))$

1-4-60
a) $\forall x(P(x) \to Q(x))$ **b)** $\exists x(R(x) \wedge \neg Q(x))$ **c)** $\exists x(R(x) \wedge \neg P(x))$

1-5-10
a) $\forall x F(x, Fred)$
b) $\forall y F(Evelyn, y)$
c) $\forall x \exists y F(x, y)$
d) $\neg\exists x \forall y F(x, y)$
e) $\forall y \exists x F(x, y)$
f) $\neg\exists x(F(x, Fred) \wedge F(x, Jerry))$
g) $\exists y_1 \exists y_2(F(Nancy, y_1) \wedge F(Nancy, y_2) \wedge y_1 \neq y_2 \wedge \forall y(F(Nancy, y) \to (y = y_1 \vee y = y_2)))$
h) $\exists y(\forall x F(x, y) \wedge \forall z(\forall x F(x, z) \to z = y))$
i) $\neg\exists x F(x, y)$
j) $\exists x \exists y(x \neq y \wedge F(x, y) \wedge \forall z((F(x, z) \wedge z \neq x) \to z = y))$

1-5-16
We let $P(s, c, m)$ be the statement that student s has class standing c and is majoring in m. The variable s ranges over students in the class, the variable c ranges over the four class standings, and the variable m ranges over all possible majors.
a) The proposition is $\exists s \exists m P(s, junior, m)$. It is true from the given information.
b) The proposition is $\forall s \exists c P(s, c, computerscience)$. This is false, since there are some mathematics majors.
c) The proposition is $\exists s \exists c \exists m P(s, c, m) \wedge (c \neq junior) \wedge (m \neq mathematics))$. This is true, since there is a sophomore majoring in computer science.
d) The proposition is $\forall s(\exists c P(s, c, computerscience) \vee \exists m P(s, sophomore, m))$. This is false, since there is a freshman mathematics major. e) The proposition is $\exists m \forall c \exists s P(s, c, m)$. This is false. It cannot be that m is mathematics, since there is no senior mathematics major, and it cannot be that m is computer science, since there is no freshman computer science major. Nor, of course, can m be any other major.

1-5-32
As we push the negation symbol toward the inside, each quantifier it passes must change its type.
a) $\forall z \exists y \exists x \neg T(x, y, z)$
b) $\forall x \forall y \neg P(x, y) \vee \exists x \exists y \neg Q(x, y)$
c) $\forall x \forall y(\neg Q(x, y) \leftrightarrow Q(y, x))$
d) $\exists y \forall x \forall z(\neg T(x, y, z) \wedge \neg Q(x, y))$

1-6-14

a) Let $c(x)$ be "x is in this class," let $r(x)$ be "x owns a red convertible," and let $t(x)$ be "x has gotten a speeding ticket."

Step	**Reason**
1. $\forall x(r(x) \to t(x))$	Hypothesis
2. $r(Linda) \to t(Linda)$	Universal instantiation using (1)
3. $r(Linda)$	Hypothesis
4. $t(Linda)$	Modus ponens using (2) and (3)
5. $c(Linda)$	Hypothesis
6. $c(Linda) \wedge t(Linda)$	Conjunction using (4) and (5)
7. $\exists x(c(x) \wedge t(x))$	Existential generalization using (6)

b) Let $r(x)$ be "x is one of the five roommates listed," let $d(x)$ be "x has taken a course in discrete mathematics," and let $a(x)$ be "x can take a course in algorithms." In what follows y represents an arbitrary person.

Step	**Reason**
1. $\forall x(r(x) \to d(x))$	Hypothesis
2. $r(y) \to d(y)$	Universal instantiation using (1)
3. $\forall x(d(x) \to a(x))$	Hypothesis
4. $d(y) \to a(y)$	Universal instantiation using (3)
5. $r(y) \to a(y)$	Hypothetical syllogism using (2) and (4)
6. $\forall x(r(x) \to a(x))$	Universal generalization using (5)

c) Let $s(x)$ be "x is a movie produced by Sayles," let $c(x)$ be "x is a movie about coal miners," and let $w(x)$ be "movie x is wonderful." In our proof, y represents an unspecified particular movie.

Step	**Reason**
1. $\exists x(s(x) \wedge c(x))$	Hypothesis
2. $s(y) \wedge c(y)$	Existential instantiation using (1)
3. $s(y)$	Simplification using (2)
4. $\forall x(s(x) \to w(x))$	Hypothesis
5. $s(y) \to w(y)$	Universal instantiation using (4)
6. $w(y)$	Modus ponens using (3) and (5)
7. $c(y)$	Simplification using (2)
8. $w(y) \wedge c(y)$	Conjunction using (6) and (7)
7. $\exists x(c(x) \wedge w(x))$	Existential generalization using (8)

d) Let $c(x)$ be "x is in this class," let $f(x)$ be "x has been to France," and let $l(x)$ be "x has visited Louvre." In our proof, y represents an unsepcified particular person.

Step	Reason
1. $\exists x(c(x) \wedge f(x))$	Hypothesis
2. $c(y)f(y)$	Existential instantiation using (1)
3. $f(y)$	Simplification using (2)
4. $c(y)$	Simplification using (2)
5. $\forall x(f(x) \to l(x))$	Hypothesis
6. $f(y) \to l(y)$	Universal instantiation using (5)
7. $l(y)$	Modus ponens using (3) and (6)
8. $c(y) \wedge l(y)$	Conjunction using (4) and (7)
9. $\exists x(c(x) \wedge l(x))$	Existential generalization using (6)

1-6-16
a) This is correct, using universal instantiation and modus tollens.
b) This is not correct. After applying universal instantiation, it contains the fallacy of denying the hypothesis.
c) After applying universal instantiation, it contains the fallacy of affirming the conclusion.
d) This is correct, using universal instantiation and modus ponens.

1-7-6
An odd number is one of the form $2n+1$, where n is an integer. We are given two odd numbers, say $2a+1$ and $2b+1$. Their product is $(2a+1)(2b+1) = 4ab+2a+2b+1 = 2(2ab+a+b+a)+1$. This last expression shows that the product is odd, since it is of the form $2n+1$, with $n = 2ab+a+b$.

1-7-8
Let $n = m^2$. If $m = 0$, then $n+2 = 2$, which is not a perfect square, so we can assume that $m \geq 1$. The smallest perfect square greater than n is $(m+1)^2$, and we have $(m+1)^2 = m^2+2m+1 = n+2m+1 > n+2 \cdot 1+1 > n+2$. Therefore $n+2$ cannot be a perfect square.

1-8-6
Because x and y are of opposite parities, we can assume without loss of generality, that x is even and y is odd. This tells us that $x = 2m$ for some integer m and $y = 2n+1$ for some integer n. Then $5x+5y = 5(2m)+5(2n+1) = 10m+10n+5 = 2[5(m+n+2)]+1$, which satisfies the definition of being an odd number.